FLUID MECHANICS

For
MECHANICAL ENGINEERING
CIVIL ENGINEERING
SYLLABUS

Fluid properties; fluid statics, manometry, buoyancy, forces on submerged bodies, stability of floating bodies; control-volume analysis of mass, momentum and energy; fluid acceleration; differential equations of continuity and momentum; Bernoulli’s equation; dimensional analysis; viscous flow of incompressible fluids, boundary layer, elementary turbulent flow, flow through pipes, head losses in pipes, bends and fittings.

Turbomachinery: Impulse and reaction principles, velocity diagrams, Pelton-wheel, Francis and Kaplan turbines.

ANALYSIS OF GATE PAPERS

<table>
<thead>
<tr>
<th>MECHANICAL</th>
<th>CIVIL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Exam Year</strong></td>
<td><strong>1 Mark Ques.</strong></td>
</tr>
<tr>
<td>2003</td>
<td>1</td>
</tr>
<tr>
<td>2004</td>
<td>2</td>
</tr>
<tr>
<td>2005</td>
<td>1</td>
</tr>
<tr>
<td>2006</td>
<td>3</td>
</tr>
<tr>
<td>2007</td>
<td>3</td>
</tr>
<tr>
<td>2008</td>
<td>1</td>
</tr>
<tr>
<td>2009</td>
<td>-</td>
</tr>
<tr>
<td>2010</td>
<td>4</td>
</tr>
<tr>
<td>2011</td>
<td>1</td>
</tr>
<tr>
<td>2012</td>
<td>3</td>
</tr>
<tr>
<td>2013</td>
<td>2</td>
</tr>
<tr>
<td>2014 Set-1</td>
<td>1</td>
</tr>
<tr>
<td>2014 Set-2</td>
<td>2</td>
</tr>
<tr>
<td>2014 Set-3</td>
<td>3</td>
</tr>
<tr>
<td>2014 Set-4</td>
<td>2</td>
</tr>
<tr>
<td>2015 Set-1</td>
<td>1</td>
</tr>
<tr>
<td>2015 Set-2</td>
<td>2</td>
</tr>
<tr>
<td>2015 Set-3</td>
<td>2</td>
</tr>
<tr>
<td>2016 Set-1</td>
<td>3</td>
</tr>
<tr>
<td>2016 Set-2</td>
<td>2</td>
</tr>
<tr>
<td>2016 Set-3</td>
<td>3</td>
</tr>
<tr>
<td>2017 Set-1</td>
<td>4</td>
</tr>
<tr>
<td>2017 Set-2</td>
<td>4</td>
</tr>
<tr>
<td>2018 Set-1</td>
<td>3</td>
</tr>
<tr>
<td>2018 Set-2</td>
<td>2</td>
</tr>
</tbody>
</table>
## CONTENTS

<table>
<thead>
<tr>
<th>Topics</th>
<th>Page No</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. BASICS OF FLUID MECHANICS</strong></td>
<td></td>
</tr>
<tr>
<td>1.1 Definition of Fluid</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Basic Equations</td>
<td>1</td>
</tr>
<tr>
<td>1.3 System and Control Volume</td>
<td>1</td>
</tr>
<tr>
<td><strong>2. PROPERTIES OF FLUIDS</strong></td>
<td></td>
</tr>
<tr>
<td>2.1 Density</td>
<td>2</td>
</tr>
<tr>
<td>2.2 Specific Gravity/Relative Density</td>
<td>2</td>
</tr>
<tr>
<td>2.3 Viscosity</td>
<td>2</td>
</tr>
<tr>
<td>2.4 Surface Tension</td>
<td>5</td>
</tr>
<tr>
<td>2.5 Capillarity</td>
<td>6</td>
</tr>
<tr>
<td>2.6 Thermo Dynamic Properties</td>
<td>7</td>
</tr>
<tr>
<td>2.7 Compressibility and Bulk Modulus</td>
<td>7</td>
</tr>
<tr>
<td>2.8 Vapor Pressure</td>
<td>8</td>
</tr>
<tr>
<td>2.9 Cavitations</td>
<td>8</td>
</tr>
<tr>
<td>Gate Questions</td>
<td>9</td>
</tr>
<tr>
<td><strong>3. PRESSURE &amp; FLUID STATICS</strong></td>
<td></td>
</tr>
<tr>
<td>3.1 Pressure</td>
<td>12</td>
</tr>
<tr>
<td>3.2 The Barometer and Atmospheric Pressure</td>
<td>13</td>
</tr>
<tr>
<td>3.3 Principles of Fluid Statics</td>
<td>14</td>
</tr>
<tr>
<td>3.4 Pressure Measurement</td>
<td>14</td>
</tr>
<tr>
<td>3.5 Hydrostatic Forces on Surfaces</td>
<td>21</td>
</tr>
<tr>
<td>3.6 Buoyancy &amp; Floatation</td>
<td>33</td>
</tr>
<tr>
<td>Gate Questions</td>
<td>40</td>
</tr>
<tr>
<td><strong>4. KINEMATICS</strong></td>
<td></td>
</tr>
<tr>
<td>4.1 Introduction</td>
<td>48</td>
</tr>
<tr>
<td>4.2 Methods of Describing Fluid Motion</td>
<td>48</td>
</tr>
<tr>
<td>4.3 Types of Fluid Flow</td>
<td>48</td>
</tr>
<tr>
<td>4.4 Continuity Equation in Three-Dimensions</td>
<td>50</td>
</tr>
<tr>
<td>4.5 Continuity Equation in One Dimension</td>
<td>51</td>
</tr>
<tr>
<td>4.6 Motion of Fluid Element</td>
<td>51</td>
</tr>
<tr>
<td>4.7 Flow Patterns</td>
<td>52</td>
</tr>
<tr>
<td>4.8 Stream Function</td>
<td>53</td>
</tr>
<tr>
<td>4.9 Velocity Potential Function</td>
<td>53</td>
</tr>
<tr>
<td>4.10 Equipotential Line</td>
<td>54</td>
</tr>
<tr>
<td>Gate Questions</td>
<td>59</td>
</tr>
<tr>
<td><strong>5. BERNOULLI’S EQUATION &amp; ITS APPLICATIONS</strong></td>
<td></td>
</tr>
<tr>
<td>5.1 Introduction</td>
<td>69</td>
</tr>
</tbody>
</table>
5.2 Euler's Equation 69
5.3 Bernoulli’s Equation 69
5.4 Application of Bernoulli’s Equation 70
5.5 Bernoulli’s Equation for Real Fluids 72
5.6 Free Liquid Jets 72

6. Dynamics of Fluid Flow
6.1 Introduction 77
6.2 Impulse Momentum Principle 77
6.3 Force Exerted by a Fluid on a Pipe Bend 77
6.4 Force Exerted by a Fluid on Vertical Stationary Plate 77
6.5 Force Exerted by a Fluid on Moving Blade 78
Gate Questions 85

7. Flow Through Conduits/Pipes
7.1 Internal Flow 93
7.2 Laminar Flow/Viscous Flow 93
7.3 Turbulent Flow in Pipes 95
7.4 Loss of Energy in Fluid Flow 98
7.5 Flow through Pipes in Series or Flow through Compound Pipes 99
7.6 Flows through Nozzles 100
Gate Questions 108

8. External Flow
8.1 Boundary Layer Formation 114
8.2 Regions of Boundary Layer 114
8.3 Boundary Layer Thickness 115
8.4 Drag Force on a Flat Plate Due to Boundary Layer 115
8.5 Boundary Condition for the Velocity Profile 116
8.6 Analysis of Turbulent Boundary Layer 117
8.7 Boundary Layer Separation 117
Gate Questions 122

9. Hydraulic Turbines
9.1 Introduction 136
9.2 Classification of Turbines 136
9.3 Turbine Efficiencies 136
9.4 Power Developed by Turbine (Euler's Equation) 137
9.5 Pelton Turbine 137
9.6 Reaction Turbine 138
9.7 Francis Turbine 140
9.8 Kaplan Turbine 140
9.9 Specific Speed Significance 141
9.10 Model Testing (Dimensionless Turbine Parameters) 141
Gate Questions 149

10. Assignment Questions 159

11. Civil Gate Questions 189
1.1 DEFINITION OF FLUID

Fluid mechanics deals with the behaviour of fluids at rest and in motion. A fluid is a substance that deforms continuously under the application of a shear (tangential) stress no matter how small the shear stress may be.

Fluids comprise the liquid and gas (or vapour) phases of the physical forms in which matter exists. The distinction between a fluid and the solid state of matter is clear if you compare fluid and solid behaviour. A solid deforms when a shear stress is applied, but its deformation does not continue to increase with time.

1.2 BASIC EQUATIONS

Analysis of any problem in fluid mechanics necessarily begins, either directly or indirectly, with statements of the basic laws governing the fluid motion. The basic laws, which are applicable to any fluid, are:

1. The conservation of mass
2. Newton’s second law of motion
3. The principle of angular momentum
4. The first law of thermodynamics
5. The second law of thermodynamics

Clearly, not all basic laws always are required to solve any one problem. On the other hand, in many problems it is necessary to bring into the analysis additional relations, in the form of equations of state or conservation equations, that describe the behaviour of physical properties of fluid under given conditions.

1.3 SYSTEM AND CONTROL VOLUME

A system is defined as a fixed, identifiable quantity of mass. The system boundaries separate the system from the surroundings.

The boundaries of the system may be fixed or movable; however, there is no mass transfer across the system boundaries. In the familiar piston–cylinder assembly the gas in the cylinder is the system. Heat and work may cross the boundaries of the system, but the quantity of the matter within the system boundaries remains fixed. There is no mass transfer across the system boundaries.

A control volume is an arbitrary volume in the space through which fluid flows. The geometric boundary of the control volume is called the control surface. The control surface may be real or imaginary; it may be at rest or in motion.
2.1 DENSITY

Density is defined as mass per unit volume and denoted by ρ. SI unit of density is Kg/m³.

\[ \text{Density}(\rho) = \frac{\text{mass of fluid}}{\text{volume of fluid}} \]

The reciprocal of density is the specific volume (\(v\)), which is defined as volume per unit mass.

SI unit of density is m³/Kg

\[ v = \frac{1}{\rho} \]

The density of a substance, in general, depends on temperature and pressure. The density of most gases is proportional to pressure and inversely proportional to temperature. Liquids and solids, on the other hand, are essentially incompressible substances, and the variation of their density with pressure is usually negligible.

2.2 SPECIFIC GRAVITY/RELATIVE DENSITY

Specific gravity or relative density of a substance is defined as the ratio of the density of a substance to the density of standard substance at a specified temperature (usually water at 4°C, for which water is 1000 kg/m³).

Specific Gravity = \( \frac{\text{Density of liquid}}{\text{Density of water}} \)

2.3 VISCOSITY

When two solid bodies in contact move relative to each other, a friction force develops at the contact surface in the direction opposite to motion. Fluid is a substance that deforms continuously under the action of shear stress. The situation is similar when a fluid moves relative to a solid or when two fluids move relative to each other. A property that represents the internal resistance of a fluid to motion is called viscosity. The force, a flowing fluid exerts on a body in the flow direction is called the drag force, and the magnitude of this force depends, in part, on viscosity.

2.3.1 VISCOUS FORCE IN LIQUIDS AND GASES

1) Molecular momentum transfer:
   In the flow of liquids and gases, molecules are free to move from one layer to another. When the velocity in the layers are different as in viscous flow, the molecules moving from the layer at lower speed to the layer at higher speed have to be accelerated. Similarly, the molecules moving from the layer at higher velocity to a layer at lower velocity, carry with them a higher value of momentum and these are to be slowed down. Thus, the molecules diffusing across layers transport a net momentum, introducing a shear stress between the layers. The force will be zero if both layers move at the same speed or if the fluid is at rest.

2) Cohesive force:
   When cohesive forces exist between atoms or molecules these forces have to be overcome, for relative motion between layers. A shear force is to be exerted to cause fluids to flow.

2.3.2 TYPES OF FLUID

Ideal fluid: Consider a fluid layer between two very large parallel plates (or equivalently, two parallel plates immersed in a large body of a fluid) separated by a
small distance \((dy)\). Now a constant tangential force \(F\) is applied to the upper plate while the lower plate is held fixed. After some time upper plate moves continuously under the influence of this force at a constant velocity \(V\). The fluid in contact with the upper plate sticks to the plate surface and moves with same velocity as that of the surface. This condition is known as no slip condition.

The shear stress \(\tau_{yx}\) acting on this fluid layer is
\[
\tau_{yx} = \frac{F}{A}
\]

Deformation or shear stain is denoted by \(d\alpha\)

Rate of shear stain or deformation is given by
\[
\frac{d\alpha}{dt} = \frac{d}{dy} \tan(d\alpha) = d\alpha = \frac{dl}{dy}
\]

\[
\frac{d\alpha}{dt} = \frac{du}{dy}
\]

Where,
\[
u\] is the velocity is \(x\) direction.

\[
\frac{du}{dy} \quad \text{is known as velocity gradient.}
\]

1) **Newtonian fluid:** Fluids for which the rate of deformation is proportional to the shear stress are called **Newtonian fluids.**

\[
\tau_{yx} \propto \text{rate of deformation or rate of shear strain.}
\]

\[
\tau = \mu \frac{d}{dt}
\]

\[
\tau = \mu \frac{du}{dy}
\]

Where, 
\(\mu\) is constant of proportionality and is known as **Dynamic viscosity.** SI unit is \(\text{kg/m-s}, \text{N. s/m}^2\) or \(\text{Pa} / \text{s}\)

**Note:**
1) \(\text{Pa}\) is the pressure unit Pascal.
2) A common viscosity unit is poise 10 Poise = 1Ns / \(\text{m}^2\)

\[
1 \text{centipoise} = 10^{-2} \text{Poise} = 10^{-3} \text{Ns} / \text{m}^2
\]
3) The viscosity of water at 20°C is 1 centipoise, and thus the unit centipoise serves as a useful reference.

2) **Non Newtonian Fluids:** Fluids in which shear stress is not directly proportional to deformation rate are known as non-Newtonian fluids. Non Newtonian fluids commonly are classified as having time independent or time dependent behavior. Numerous equations have been proposed to model the observed relations between \(\tau\) and \(\frac{du}{dy}\) for time-independent fluids.

\[
\tau = K \left( \frac{du}{dy} \right)^n
\]

Where,
\(n\) is called the flow behavior index

\(k\) the consistency index.

\[
\tau = k \left( \frac{du}{dy} \right)^{n-1} \frac{du}{dy} = \eta \frac{du}{dy}
\]

The term \(\eta = k \left( \frac{du}{dy} \right)^{n-1}\) is referred to as the apparent viscosity.

Fluids in which the apparent viscosity decreases with increasing deformation rate \((n<1)\) are called pseudo plastic (or shear thinning) fluids. Most non-Newtonian fluids fall into this group; Examples include polymer solutions, colloidal suspensions, and paper pulp in water. If the apparent viscosity increases with increasing deformation rate \((n>1)\) the fluid is termed dilatants (or shear thickening). Suspension of starch and of sand are examples of dilatants fluids.
A “fluid” that behaves as a solid until a minimum yield stress, $\tau$, is exceeded and subsequently exhibits a linear relation between stress and rate of deformation is referred to as an ideal or Bingham plastic. Thixotropic fluids show a decrease in $\eta$ with time under a constant applied shear stress. Rheopectic fluids show an increase in $\eta$ with time. After deformation some fluids partially return to their original shape when the applied stress is released; such fluids are called viscoelastic.

2.3.3 KINEMATIC VISCOSITY

Kinematic viscosity is defined as the ratio between dynamic viscosity and density denoted by $'\nu'$
$$\nu = \frac{\mu}{\rho}$$

The unit in SI system is m$^2$/s. Stoke is CGS unit of kinematic viscosity given by
1 (cm$^2$/s) = $10^{-4}$ m$^2$/s.
1 centistoke = $10^{-6}$ m$^2$/s.

Kinematic viscosity gives the rate of momentum flux or momentum diffusivity. For liquids and gases absolute (dynamic) viscosity is not influenced significantly by pressure. But kinematic viscosity of gases is influenced by pressure due to change in density.

2.3.4 EFFECT OF TEMPERATURE ON VISCOSITY

1) **Liquids:** In case of liquids the viscosity force is mainly due to cohesive force. The cohesive force decreases. So, viscosity of liquids decreases when temperature increases. The relation of viscosity with temperature is given by
$$\mu = \mu_0 \left( \frac{1}{1 + \alpha T + \beta T^2} \right)$$

2) **Gases:** In the case of gases, the contribution to viscosity is more due to momentum transfer. As temperature increases, more molecules cross over with higher momentum differences. Hence, in the case of gases, viscosity increases with temperature.
$$\mu = \mu_0 + \alpha T + \beta T^2$$

where,
$$\mu = \text{Viscosity at } T \text{ in poise}$$
$$\mu_0 = \text{Viscosity at } 0 \text{ in poise}$$
$\alpha, \beta =$ are constants for liquid and gas

**SOLVED EXAMPLES**

Example:
‘An infinite plate is moved with a velocity of 0.3m/s over a second plate on a layer of liquid for small gap width $d=0.3$mm, assume a linear velocity distribution the liquid viscosity is $30.65 \times 10^{-3}$kg/m/s and S.G is 0.88.

a) Calculate kinematic viscosity
b) The shear stress on the lower plate

**Solution:**

a) $V = \frac{\mu}{\rho}$

$$= 0.65 \times 10^{-3} \frac{\text{kg}}{\text{m.s}} \times \frac{\text{m}^3}{(0.88 \times 1000) \text{kg}}$$

$$= 7.39 \times 10^{-7} \text{ m}^2 / \text{s}$$

b) $\tau_{\text{lower}} = \mu \left( \frac{u}{d} \right)$
\[
\tau_{\text{lower}} = 0.65 \text{Pa}
\]

Direction of shear stress on lower plate is \( \tau_{\text{lower}} \)

**Example:**

Calculate the dynamic viscosity of oil, which is used for lubrication b/w a square of size \( 0.8 \times 0.8 \text{ m}^2 \) and an inclined plane with angle of inclination 30°, \( \text{wt} = 300 \text{ N} \), slides down the inclined plane with a uniform velocity of 0.3 m/s. the thickness of oil film is 1.5 mm.

**Solution:**

\[
\begin{align*}
\text{W sin } \theta &= F_{\text{oil}} \\
\frac{F_{\text{oil}}}{F_{\text{contact}}} &= \mu \frac{\Delta u}{\Delta y} \\
F_{\text{oil}} &= \mu \frac{u}{y} \\
\mu &= \frac{F_{\text{oil}}}{A \cdot y} \\
\mu &= W \sin \theta \cdot y \\
\mu &= \frac{300 \sin 30 \times 1.5 \times 10^{-3}}{0.64 \times 0.3} = 0.117 \times 10^{\text{Poise}} \\
\mu &= 1.17 \text{Poise}
\end{align*}
\]

## 2.4 SURFACE TENSION

Consider two liquid molecules, one at the surface and one deep within the liquid body. The attractive forces applied on the interior molecule by the surrounding molecules balance each other because of symmetry. But the attractive forces acting on the surface molecule are not symmetric, and the attractive forces applied by the gas molecules above are usually very small. Therefore, there is net attractive force acting on the molecule at the surface of the liquid, which tends to pull the molecules on the surface toward the interior of the liquid. This force is balanced by the repulsive forces from the molecules below the surface that are being compressed. The resulting compression effect causes the liquid to minimize its surface area. This is the reason for the tendency of the liquid droplets to attain a spherical shape, which has the minimum surface area for a given volume.

The surface of the liquid acts like a stretched elastic membrane under tension. The pulling force that causes this tension acts parallel to the surface and is due to the attractive forces between the molecules of the liquid. The magnitude of this force per unit length is called surface tension and is usually expressed in the unit N/m.

Surface tension is also defined as the surface energy per unit surface area or work that needs to be done to increase the surface area of the liquid by a unit amount. Surface tension is a binary property of the liquid & gas or two liquids. The surface tension of air and water at 20°c is about 0.73 N/m.

If \( F \) is the tensile force on the surface, \( L \) is the length of the surface. Surface tension is given by

\[
\sigma = \frac{F}{L} \quad \text{or} \quad \sigma = \frac{E_{\text{surface}}}{\text{surface Area}}
\]

### 2.4.1 SURFACE TENSION ON LIQUID DROPLET

Let \( \sigma \) is surface tension

R is radius of droplet
A_p is area of projection

1) Force due to difference in pressure inside & outside the liquid drop
   \[ \Delta P A_p = \Delta P \pi R^2 \]  \[ \quad \text{(i)} \]

2) Tensile force due to surface tension
   \[ = \sigma \times \text{circumference} \]
   \[ = \sigma \times 2\pi R \]  \[ \quad \text{(ii)} \]

Under equilibrium condition, these 2 forces will be equal

\[ \therefore \Delta P = \frac{2\sigma}{R} \]

### 2.4.2 SURFACE TENSION ON A SOAP BUBBLE

1) Force due to pressure inside the liquid drop
   \[ = \Delta P A_p \]
   \[ = \Delta P \pi R^2 \]  \[ \quad \text{(i)} \]

2) Tensile force due to surface tension
   \[ F_{\text{surface}} = 2\pi R \sigma \]
   \[ F_{\text{surface}} = 2\pi (R + t) \sigma \]  \[ \quad \text{(ii)} \]

Equating the forces, we get

\[ \Delta P \pi t^2 = \sigma 2\pi (R + t) + \sigma 2\pi R \]

Assuming thickness is very small

\[ T << R \]

\[ \Delta P = \frac{4\sigma}{R} \]

### 2.4.3 SURFACE TENSION ON A LIGHT JET

1) Force due to pressure inside & outside the liquid jet
   \[ = \Delta P L (2R) \]
   \[ \quad \text{(i)} \]

2) Tensile force due to surface tension
   \[ = 2L \sigma \]
   \[ \quad \text{(ii)} \]

Equating the forces, we get

\[ \Delta P = \frac{\sigma}{R} \]

### 2.5 CAPILLARITY

Another consequence of surface tension is the capillary effect, which is the rise or fall of a liquid in a small-diameter tube inserted into the liquid. Such narrow tubes or confined flow channels are called **capillaries**. The curved free surface of a liquid in a capillary tube is called the **meniscus**. It is commonly observed that water in a glass container curves up slightly at the edges where it touches the glass surface; but the opposite occurs for mercury: it curves down at the edges. The strength of the capillary effect is quantified by the contact (or wetting) angle \( \beta \), defined as the angle that the tangent to the liquid surface makes with the solid surface at the point of contact. The surface tension force acts along this tangent line toward the solid surface.
1) Force due to surface tension
\[ F_y = \sigma \pi r \cos \beta \]
\[ F_x = 0 \]

2) Vertical force is responsible for lifting the liquid in capillary
\[ F_y = \rho \pi r^2 h \cdot g \]
Equating the vertical forces, we get
\[ 2\pi \sigma \cos \beta = \rho \pi r^2 h \cdot g \]
\[ h = \frac{2\sigma \cos \beta}{\rho g r} \]

Note:
Same expression is used for capillary fall. Angle, \( \beta = 0 \) for glass tube & water. \( \beta = 128^\circ \) for glass tube & mercury.

2.6 THERMO DYNAMIC PROPERTIES

2.6.1 IDEAL GAS EQUATION

\[ PV = nRT \]
Where,
\( P \) is pressure in Pa
\( V \) is volume in m\(^3\)
\( n \) is moles of gas
\( R \) is universal gas constant (8.314 \( \text{KJ/mole K} \))

\[ PV = \left( \frac{M}{M_w} \right) RT \]
\[ \Rightarrow PV = m \left( \frac{R}{M_w} \right) T \]
\[ \Rightarrow PV = m (R) T \]
\[ \Rightarrow P = \frac{m}{V} R' T \]
\[ \Rightarrow P = \rho R'' T \]

Where,
\( R' \) is characteristic gas constant & for air
\[ R' = 287 \frac{\text{J}}{\text{kg} \cdot \text{K}} \]
\( M_w \) is molecular wt of gas
\( \rho \) is density of gas

2.6.2 THERMODYNAMIC PROCESS

a) Isothermal:
Constant Temperature
\[ PV = mRT \]
\( T = \text{const} \)
\[ PV = \text{const} \]

b) Adiabatic Process:
No heat transfer takes place
\[ \frac{P}{\rho'^{\gamma}} = \text{const} \]
\( \gamma \) is ratio of specific heat
\[ \gamma = \frac{C_p}{C_v} \]
\( \gamma = 1.4 \) for air

c) Isobaric process:
Constant pressure process
\[ V = \text{const} \]
\[ \frac{P}{T} = \text{const} \]

d) Isochoric process:
Constant volume process
\[ V = \text{const} \]
\[ \frac{P}{T} = \text{const} \]

2.7 COMPRESSIBILITY AND BULK MODULUS

It is the measure of volume change under the action of external force.
Let the volume of gas decrease from \( V \) to \((v-dv)\), the pressure is increased from \( P \) to \( P+dP \)

Increase in pressure
\[ = dp \text{ kgf / m}^2 \]

Decrease in volume = \( dv \)

Volumetric strain = \( \frac{-dv}{v} \)

Bulk Modulus (\( k \)) = \( \frac{dp}{dv/v} \)

Compressibility = \( \frac{1}{k} \)

Relationship b/w bulk modulus and pressure (\( p \)) for a gas undergoing compression process:

a) For Isothermal process:
\[ \frac{P}{\rho} = \text{const} \]
for closed system.

\( P \) by taking log
\[ \ln P + \ln V = \ln(\text{const}) \]

By differentiating
\[ \frac{dp}{p} + \frac{dV}{V} = 0 \]
\[ -\frac{dp}{dv/v} = p \]
\[ k = p \]

b) Adiabatic Process:
\[ PV^\gamma = \text{const} \]
by taking log both side
\[ \ln P + \gamma \ln V = \ln \text{const} \]

by differentiating
\[ \frac{dp}{p} + \gamma \frac{dV}{V} = 0 \]
\[ -\frac{dp}{dv/v} = \gamma P \]
\[ k = \gamma P \]

2.9 CAVITATIONS

The liquid pressure in liquid-flow systems drops below the vapour pressure at some locations, results in vaporization of liquid. For example, water at 10°C will convert into vapour and form bubbles at locations (such as the tip regions of impellers or suction sides of pumps) where the pressure drops below 1.23 kPa. The vapour bubbles (called cavitation bubbles since they form “cavities” in the liquid) collapse as they are swept away from the low pressure regions, generating highly destructive, extremely high-pressure waves. This phenomenon, which is a common cause for drop in performance and even the erosion of impeller blades, is called cavitation, and it is an important consideration in the design of hydraulic turbines and pumps.

2.8 VAPOR PRESSURE

At a given pressure, the temperature at which a pure substance changes phase is called the saturation temperature \( T_{\text{sat}} \). Likewise, at a given temperature, the pressure at which a pure substance changes phase is called the saturation pressure \( P_{\text{sat}} \). The vapour pressure \( P_v \) of a pure substance is defined as the pressure exerted by its vapour in phase equilibrium with its liquid at a given temperature. \( P_v \) is a property of the pure substance, and turns out to be identical to the saturation pressure \( P_{\text{sat}} \) of the liquid (\( P_v = P_{\text{sat}} \)).
Q.1  The SI unit of kinematic viscosity (v) is
   a) m²/s  b) kg/m-s  c) m/s²  d) m³/ s²  
   [GATE-2001]

Q.2  For a Newtonian fluid
   a) Shear stress is proportional to shear strain
   b) Rate of shear stress is proportional to shear strain
   c) Shear stress is proportional to rate of shear strain
   d) Rate of shear stress is proportional to rate of shear strain
   [GATE-2006]

Q.3  Assuming constant temperature condition and air to be an ideal gas, the variation in atmospheric pressure with height calculated from fluid statics is
   a) linear  b) exponential  c) quadratic  d) cubic
   [GATE-2016(2)]

Q.4  A solid block of 2.0 kg mass slides steadily at a velocity V along a vertical wall as shown in the figure below. A thin oil film of thickness h = 0.15 mm provides lubrication between the block and the wall. The surface area of the face of the block in contact with the oil film is 0.04m². The velocity distribution within the oil film gap is linear as shown in the figure. Take dynamic viscosity of oil as $7 \times 10^{-3}$ P-s and acceleration due to gravity as $10 \text{ m/s}^2$. Neglect weight of the oil. The terminal velocity V (in m/s) of the block is _________ (correct to one decimal place).
**ANSWER KEY:**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(c)</td>
<td>(b)</td>
<td>10.71</td>
</tr>
</tbody>
</table>
From the Newton’s law of Viscosity, the shear stress (τ) is directly proportional to the rate of shear strain (du/dy).

So, \( \tau \propto \frac{du}{dy} \) or \( \tau = \mu \frac{du}{dy} \)

Where, \( \mu \) = constant of proportionality and it is known as coefficient of viscosity

Terminal velocity is reached when weight force of the block is balanced by the viscous drag forces due to shear stress generated in the fluid-solid interface.

\[ \therefore \tau A = W \]

Using Newton's law of viscosity

\[ \tau = \mu \frac{du}{dy} = \mu \frac{V}{h} \] (Q linear profile)

\[ \Rightarrow \mu \frac{V}{h} A = W \Rightarrow V = \frac{Wh}{\mu A} = \frac{mgh}{\mu A} \]

\[ = \frac{2 \times 10 \times 0.15 \times 10^{-3}}{7 \times 10^{-3} \times 0.04} = 10.714 \text{ m/s} \]